Supporting Information for "Ice-floe mechanics and pressure ridging in sea ice"

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Introduction

In this document we provide a full description of the two numerical models deployed in this study. Further results are included as supplementary figures. A summary table lists simulation parameters.

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Text S1. Lagrangian modeling of compressive interaction between two ice floes

In order to investigate the mechanical interaction between ice floes, we design a Lagrangian numerical model based on the bonded discrete-element method [e.g., *Cundall and Strack*, 1979; *Potyondy and Cundall*, 2004; *Damsgaard et al.*, 2018; *Damsgaard*, 2018]. Non-bonded particles interact with a cohesion-less, elastic, and Coulomb-frictional rheology described in *Damsgaard et al.* [2018]. Bonded particles interact with elastic-plastic mechanics based on beam theory, after the 2D formulation by *Potyondy and Cundall* [2004]. The beam formulation resists relative rotation, shear, and tension between the particles. The tensile stress on a bond is limited by the ultimate tensile strength (σ_uts):

$$\sigma_\mathrm{uts}^{ij} > \frac{||\boldsymbol{f}_\mathrm{n}^{ij}||}{A^{ij}} + \frac{|M_\mathrm{t}^{ij}|R^{ij}}{I^{ij}}, \quad (1)$$

where M_t is the bending momentum on the bond:

$$M_\mathrm{t}^{ij} = \frac{k_\mathrm{n}^{ij} R^{ij}}{A^{ij}(r^i + r^j) I^{ij} \theta_\mathrm{t}^{ij}}. \quad (2)$$

I^{ij} is the particle-pair moment of inertia, approximated as,

$$I^{ij} = \frac{2}{3} R_{ij}^3 \min(h_i, h_j), \quad (3)$$

and θ_t is the total relative rotation distance of the contact ($\theta_\mathrm{t}^{ij} = \int_t (\omega^j - \omega^i) dt$). The bond can also fail if shear stress exceeds the shear strength σ_s:

$$\sigma_\mathrm{s}^{ij} > \frac{||\boldsymbol{f}_\mathrm{t}^{ij}||}{A^{ij}} \quad (4)$$

If the bond stresses (right-hand sides of Eqs. 1 and 4) exceed the prescribed strengths (σ_uts and σ_s), the bond breaks and is no longer enforced. Bonds do not re-form in these simulations, except when simulating instant refreezing (Fig. S2).

Text S2. Plan-view ridging parameterization for larger-scale particle models

The ice-floe interactions transition from an elastic and reversible pre-failure state, to a ridged or rafting post-failure state when mechanical failure and vertical redistribution of the ice mass changes the physics of interaction (Fig. 2). In the following, we describe how the interaction of plan-view particles of ice-floe size is parameterized to include the ridging dynamics and rheological changes observed from the previous compressional experiments.

S2.1 Pre-failure contact mechanics

In the stage before compressive failure occurs, the contact rheology is parameterized from linear elasticity and Coulomb friction [*Damsgaard et al.*, 2018]. The contact-normal force $\boldsymbol{f}_{\mathrm{n}}$ is given by:

$$\boldsymbol{f}_{\mathrm{n}}^{ij} = -A^{ij} E^{ij} \boldsymbol{\delta}_{\mathrm{n}}^{ij} \tag{5}$$

The contact cross-sectional area between the cylindrical elements is defined as $A^{ij} = R^{ij} \min\left(h^i, h^j\right)$, where $R^{ij} = 2 r^i r^j \left(r^i + r^j\right)^{-1}$ is the geometrical mean of the radii (Fig. 1a). E^{ij} is Young's modulus (the elastic modulus) for the contact. The contact-tangential (parallel) force $\boldsymbol{f}_{\mathrm{t}}$ is defined as,

$$\boldsymbol{f}_{\mathrm{t}}^{ij} = -\frac{E^{ij} A^{ij}}{R^{ij}} \frac{2(1 - (\nu^{ij})^2)}{(2 - \nu^{ij})(1 + \nu^{ij})} \boldsymbol{\delta}_{\mathrm{t}}^{ij} \tag{6}$$

where $\boldsymbol{\delta}_{\mathrm{t}}^{ij}$ is the tangential displacement vector on the contact interface. This vector is incrementally calculated and corrected for contact rotation [*Damsgaard et al.*, 2018]. The magnitude of the contact-tangential force is limited by Coulomb friction:

$$||\boldsymbol{f}_{\mathrm{t}}^{ij}|| \leq \mu^{ij} ||\boldsymbol{f}_{\mathrm{n}}^{ij}|| \tag{7}$$

S2.2 Criteria for compressive failure

We find that the compressive failure limit in the detailed two-floe compressional experiments (Fig. 3) is well described by a relationship of the form,

$$||\boldsymbol{f}_\mathrm{n}^{ij} + \boldsymbol{f}_\mathrm{t}^{ij}|| \leq \min\left(K_\mathrm{Ic}^i, K_\mathrm{Ic}^j\right) \min\left(h^i, h^j\right)^{3/2} \tag{8}$$

where h is the ice-floe thickness and K_Ic is the fracture toughness (units Pa m$^{1/2}$, or N m$^{-3/2}$), characterizing the resistance to brittle failure. Note that the orthogonal normal ($\boldsymbol{f}_\mathrm{n}$) and tangential contact forces ($\boldsymbol{f}_\mathrm{t}$) can both contribute to the compressive stress on the contact. The 3/2-order dependency between thickness and strength is consistent with some previous parameterizations of ridging failure [e.g., *Rothrock*, 1975; *Hopkins*, 1998], but not the commonly used linear relationship [e.g., *Hibler*, 1979].

S2.3 Post-failure contact mechanics

After compressive failure has occurred (Eq. 8), the ice-floe contact is marked as actively ridging. The previous interaction mechanics (Eq. 5 to 7) are replaced with the parameterized ridging physics. After failure the ice floes are assumed to undergo stacking as a means of vertical rearrangement (Fig. 1b). Sliding friction along the sub-horizontal contact interface governs the mechanics in the post-failure state. The normal stress on the contact interface is determined by the hydrostatic response due to density differences and buoyancy:

$$\boldsymbol{\sigma}_\mathrm{n}^{ij} = (\rho_\mathrm{w} - \rho_\mathrm{i})(h_i + h_j)\boldsymbol{g}, \tag{9}$$

where ρ_w and ρ_i are the densities of water and ice, respectively, and $\boldsymbol{g}$ is the gravitational acceleration. The interficial tangential stress $\boldsymbol{\sigma}_\mathrm{t}$ is sub-horizontal, and is determined by the horizontal sliding distance $\boldsymbol{\delta}_\text{s}$, the contact stiffness k_t, and the interface area A (Fig. 1b):

$$\boldsymbol{\sigma}_\text{t}^{ij} = -k_\text{t}\boldsymbol{\delta}_\text{s}^{ij} A_{ij}^{-1} \tag{10}$$

We uphold the Coulomb-frictional limit on the sliding interface:

$$||\boldsymbol{\sigma}_\text{t}^{ij}|| \leq \mu^{ij} ||\boldsymbol{\sigma}_\text{n}^{ij}||, \tag{11}$$

and excess elastic energy is stored as frictional heat loss. Increases in contact strength by freezing can be added to the right-hand side of the above equation through a time and temperature-dependent cohesion term, but is not included here.

The normal and tangential forces on the particles are found by decomposing the tangential stress according to the contact orientation:

$$\boldsymbol{f}_\text{n}^{ij} = (\boldsymbol{\sigma}_\text{t}^{ij} \cdot \hat{\boldsymbol{n}}^{ij}) A^{ij} \tag{12}$$

$$\boldsymbol{f}_\text{t}^{ij} = (\boldsymbol{\sigma}_\text{t}^{ij} \cdot \hat{\boldsymbol{t}}^{ij}) A^{ij}, \tag{13}$$

where $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{t}}$ are unit-length normal and tangential vectors for the i and j particle pair. The forces grow non-linearly with increasing contact area during compression (Fig. 1b).

Table S1. Simulation parameters for ice-floe compression tests.

Parameter	Symbol	Value
Granular.jl software version	—	v0.3.4[a]
Ice particle radii	r	0.01 m
Young's modulus	E	2×10^7 Pa
Poisson's ratio	ν	0.285
Coulomb friction coefficient	μ	0.3
Maximum tensile strength	σ_c	400 kPa
Maximum bond shear strength	σ_s	200 kPa
Compressive velocity	c_v	$[0.05, 0.10, 0.2]$ m/s
Ice particle density	ρ_i	934 kg/m^3
Water density	ρ_w	1000 kg/m^3
Thickness in no. of grains for left ice floe	$n_{y,1}$	$[3, 5, 7, 9, 11, 13, 15, 17, 19, 21]$
Thickness in no. of grains for right ice floe	$n_{y,2}$	$[3, 5, 7, 9, 11, 13, 15, 17, 19, 21]$
Length in no. of grains for left ice floe	$n_{x,1}$	100
Length in no. of grains for right ice floe	$n_{x,2}$	100
Gravitational acceleration	g_z	-9.8 m/s^2
Numerical time step length	Δt	8.48×10^{-6} s
Simulation length	t_total	$\frac{n_{x,1}+n_{x,2}}{2}\frac{2r}{c_\mathrm{v}}$

[a] See *Damsgaard* [2018] for DOI and downloadable snapshot.